

ESTIMATE OF CEASELESS SIZE NEEDED FOR TOTAL FACEBOOK COVERAGE

Ceaseless is a Facebook application that encourages users to pray for their Facebook friends. Here we will make a rough estimate of how many Ceaseless users there would need to be for each user of Facebook to have at least one Ceaseless user as a Facebook friend. When the number of Ceaseless users reaches this size we may expect that all, or nearly all, Facebook users will have the possibility of being covered in prayer through the Ceaseless application.

To begin, let,

$$\begin{aligned}C &= \text{Ceaseless users} \\F &= \text{Facebook users} \\|F| &= N, |C| = n, C \subset F\end{aligned}$$

We will assume for the sake of this calculation that each Facebook user has α friends which are randomly distributed within F . For a member x of F , the probability $P(x \text{ has no friends in } C)$ is,

$$\begin{aligned}P_0 &= \left(1 - \frac{n}{N}\right)^\alpha = \left(1 - \frac{n}{N}\right)^{N(\alpha/N)} \approx e^{-\alpha n/N} \text{ when } N \gg 0 \\&\text{(because } \lim_{N \rightarrow \infty} \left(1 + \frac{t}{N}\right)^N = e^t\end{aligned}$$

Then,

$$\begin{aligned}1 - P_0 &= P(x \text{ has a friend in } C) \\E(\text{FBmembers with a friend in } C) &= N(1 - P_0) \approx N(1 - e^{-\alpha n/N})\end{aligned}$$

We may expect that all Facebook users have at least one Ceaseless user as a friend with a 50% certainty approximately when,

$$\begin{aligned}
(1 - P_0)^N &= 1/2 \\
(1 - e^{-\alpha n/N}) &= (1/2)^{1/N} \\
e^{-\alpha n/N} &= 1 - (1/2)^{1/N} \\
e^{-\alpha n/N} &\approx \frac{\ln 2}{N} \\
\frac{\alpha n}{N} &\approx \ln \left(\frac{N}{\ln 2} \right) \\
n &\approx \frac{N}{\alpha} \ln \left(\frac{N}{\ln 2} \right)
\end{aligned}$$

Based on the above estimate for $E(\text{FBmembers with a friend in } C)$ we can also answer the question of approximately how many Ceaseless users would be needed to get r prayer coverage of Facebook.

$$\begin{aligned}
N(1 - e^{-\alpha n/N}) &= rN \\
1 - e^{-\alpha n/N} &= r \\
e^{-\alpha n/N} &= 1 - r \\
-\alpha n/N &= \ln(1 - r) \\
n &= \frac{N}{\alpha} \ln(1 - r)^{-1} \\
n &= \frac{N}{\alpha} \ln \left(\frac{1}{1 - r} \right)
\end{aligned}$$

Based on these calculations and the reasonable value $N \approx 1.31$ billion and $\alpha \approx 130$, we can estimate how many people need to use Ceaseless to cover all Facebook users in personal prayer.[1]

$$\begin{aligned}
n &\approx \frac{N}{\alpha} \ln \left(\frac{N}{\ln 2} \right) \\
&\approx \frac{1.31 \times 10^9}{130} \ln \left(\frac{1.31 \times 10^9}{\ln 2} \right) \\
&\approx 215 \text{ million}
\end{aligned}$$

To have 99% coverage of F the number of Ceaseless users would need to be,

$$\begin{aligned}n &\approx \frac{N}{\alpha} \ln \left(\frac{1}{1-r} \right) \\ &\approx \frac{1.31 \times 10^9}{130} \ln \left(\frac{1}{1-.99} \right) \\ &\approx 46.4 \text{ million}\end{aligned}$$

Finally, it should be noted that since connections on Facebook are neither random nor evenly distributed, the values calculated here serve only as rough estimates and the actual values in question may be somewhat significantly different.

REFERENCES

- [1] Statistic Brain. Facebook statistics, July 2014. URL <http://www.statisticbrain.com/facebook-statistics/>.